Ready-Ed Publications

For 10+ years

PHOTOCOPY MASTERS

Maths Investigations

Structured and challenging concepts to stimulate advanced mathematical thinking in 10-12 year old students.

SAMPLE

By Susan Levy

Contents

SAMPLE

Using this book in the classroom

The activities in this book have been structured to provide materials that are appropriate to the needs and abilities of a range of abilities in the mathematics class. In this book this is done through the use of CONTENT LEVELS which are an adaptation of a Taxonomy of Educational Objectives as put forward by Benjamin Bloom (1950's) and later revised and modified by Krathwohl and Anderson (2001).

Content Level	Code Used	What It Means	What The Student Does
1	*	FINDING OUT - Recalling data; showing understanding through restating or extending ideas.	Answers factual questions; interprets information; describes or illustrates events. To do this student might be involved in writing, listing, drawing, etc.
2	**	INFORMATION USING - Using content in such a way that new material is applied to, or analysed, in another situation.	Problem solving based on knowledge gained. Children might be required to do tasks that ask them to show, compare, investigate, work out or group.
3	***	CREATING/EVALUATING - Putting together ideas to develop new products or concepts; making judgements based on the content base encountered in levels 1 & 2.	Children use their extended knowledge base as the springboard for activities involving inventing, planning, constructing, imagining, improving, selecting, arguing, evaluating or recommending.

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Before starting ...

Decide on the approach to the theme that is most suitable for your students. This may be:

- ❑ Teacher directed with the target group/individual completing teacher selected sheets at a specified time. (Teacher records progress.)
- ❑ Student directed with students working through materials at their own pace at a specified time. (Student records progress.)
- ❑ As an interest based approach with students working on materials at unspecified times.
- ❑ (Student monitored and recorded.)
- ❑ As supplementary materials to a unit of study.

Outline procedural steps for the approach to be used with materials:

- ❑ What will the end-product be? (A booklet, a display, etc.?)
- ❑ How can students work on the contents? (Individually, small groups?)
- ❑ From where can further research sources be obtained?
- ❑ What people or organisations might be able to assist with the study?
- ❑ How will the materials be implemented in the classroom? (Distribution of materials, times when work to be done, extra paper, art materials, procedures for recording, etc.)

The structure of this book

At this stage of their mathematics education students should be required to investigate concepts in considerable depth. Accordingly some activities in this book stretch over several pages perhaps requiring students to carry an idea over from day to day as they move through a particular unit. This idea of mathematical investigations is intended to provide challenge for students, to encourage them to look at the real life aspects of mathematics and to stimulate them to go to even further depths than the text provides for.

The Content Levels described above are indicated in the book with the use of the terms * Finding out, ** Using your skills; and *** Taking it further, with the asterisks denoting the level of activity as in the grid.

Birthdays And Other Days cont.

Step 3.	Subtract 1 from the year and divide by 4.	1985 - 1	=	1984	
	Ignore any remainder.	1984 ÷ 4	=	496	
Step 4.	Add the answer			496	
	and the year			1985	
	and the total days.			157	
				2638	
Step 5.	Subtract 15	2638 - 15	=	2623	
	and divide by 7.	2623 ÷ 7	=	374	remainder 5

The remainder gives the day of the week:

Saturday	0
Sunday	1
Monday	2
Tuesday	3
Wednesday	4
Thursday	5 (e.g. 6 June 1985 was a Thursday.)
Friday	6

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** Let's work out your birthday.

Write your date of birth here: _____

Was it a leap year? YES / NO

Step 1. Write down the year. _____

Step 2. Add up the days. _____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Step 3. Subtract 1 from the year and divide by 4 _____ - 1 = _____

_____ ÷ 4 = _____

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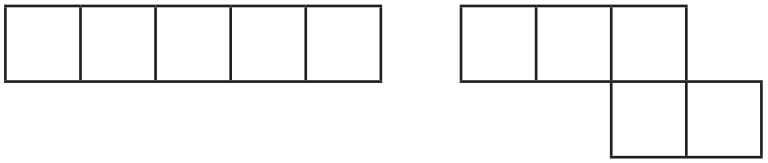
Pentominoes

Investigation 5

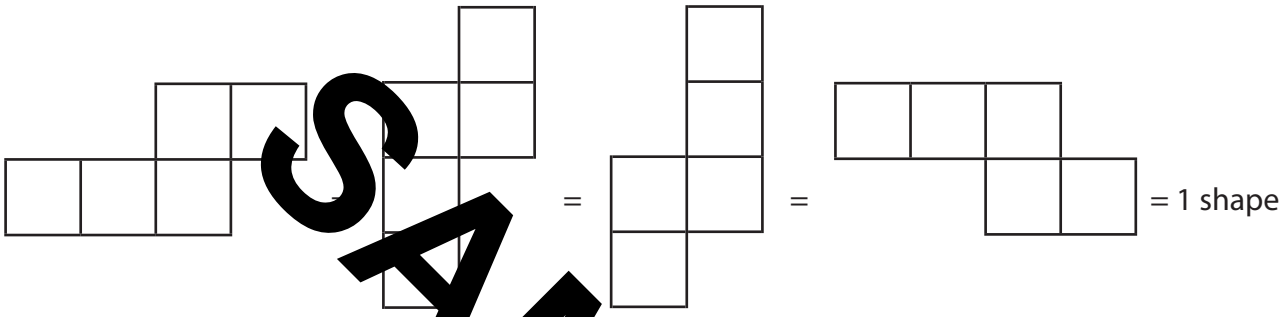
* Finding Out

You will need a sheet of squared paper and a pair of scissors for this activity.

Five squares fitted together edge-to-edge make a **pentomino**. Edges must fit exactly.

Some examples of pentominoes are 

There are twelve different pentomino shapes possible. (Turning a shape over still only counts as one.)

i.e.  = 1 shape

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❑ Using squared paper, see if you can draw them. Once you have drawn them, copy them onto firm card and cut them out.

** Using Your Skills

❑ Now here's a challenge for you. Try to fit the twelve shapes together to form a 10 × 6 rectangle. It will take patience but it can be done.

Extra Challenge:

The 12 pentominoes can also be fitted exactly into rectangles of 12 × 5, 15 × 4 and 20 × 3 squares. There are thousands of solutions to each rectangle, but you will be doing really well if you can come up with one of each. See how you go.

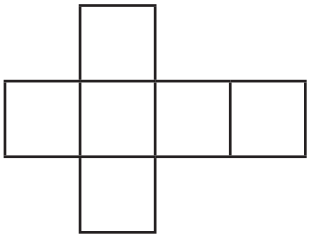
Extra-Super Challenge:

Six squares fitted together edge-to-edge form a hexomino. There are thirty five of these. Can you find them all? You will need to work to a pattern and record your results carefully.
It might help if you work with a partner for this one. Good luck!

Double-Extra-Super Challenge:

Many of the hexominoes can be folded to form a cube or box.

This is an example of one which does.
Work out how many hexominoes can fold into cubes.
You may be able to do this without actually cutting and folding them if you can work out how the edges touch.

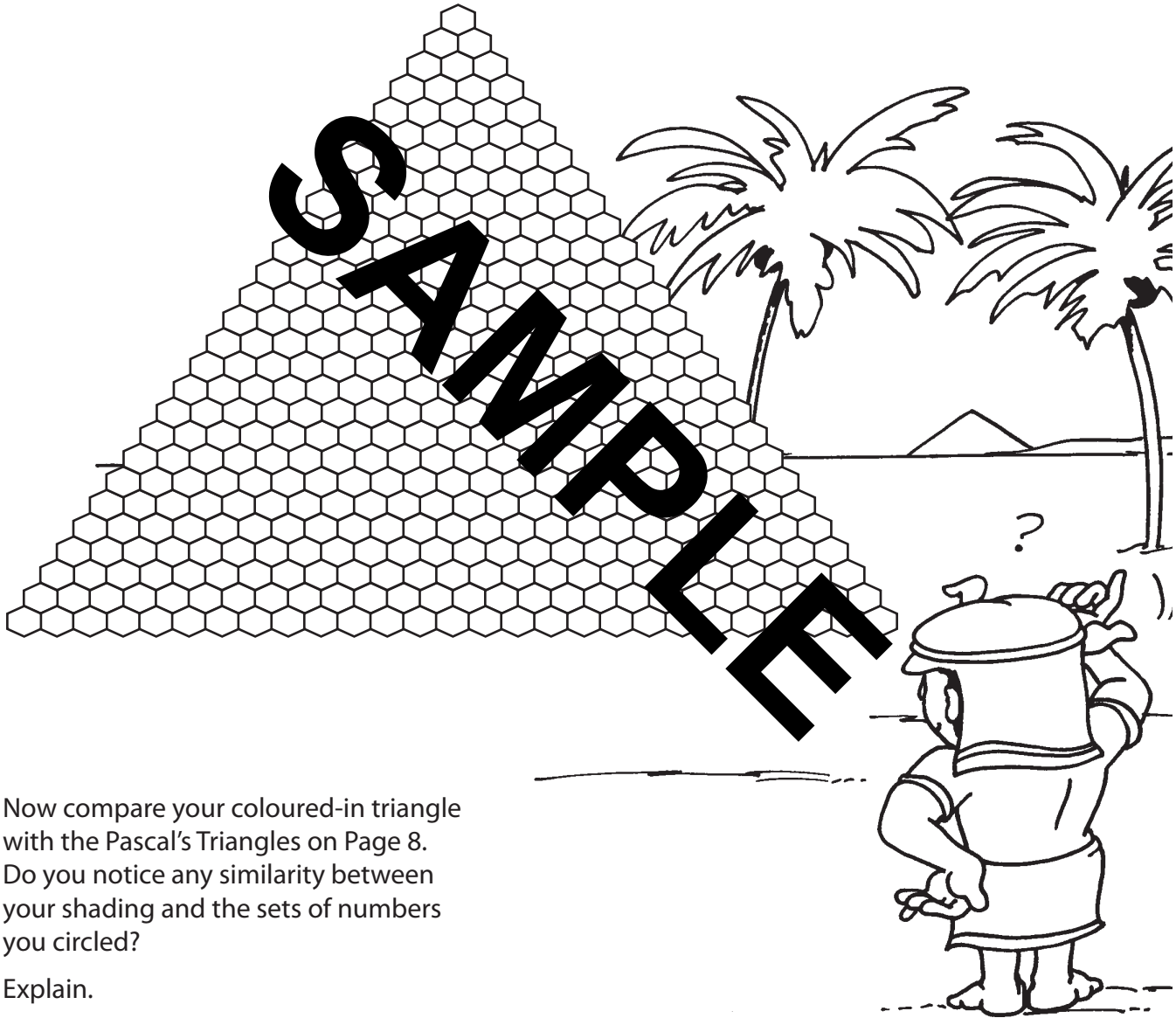


*** Taking It Further

Here is the Sierpinski Triangle again. This time it is made up of regular hexagons.

We are going to colour the hexagons according to this set of rules:

- ❑ all cells on the end of a row are coloured in;
- ❑ if the two cells above a cell are the same colour (both black or both white) we leave that cell white. Otherwise colour it.



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Now compare your coloured-in triangle with the Pascal's Triangles on Page 8. Do you notice any similarity between your shading and the sets of numbers you circled?

Explain.

Playing Cribbage 1 cont.

If you have a Jack in your hand, and the starter is the same suit as the Jack, you can add 1 point. (This is called 1 for His Nob.) If the Dealer turns up a Jack as the starter, (s)he adds 2 points (2 for His Heels). This is one of the benefits of being Dealer.

An example		4♥, 5♣, 6♥, Q♦		Starter is J♣.
	15s	4♥, 5♣, 6♥	2	Dealer would also add 2 for
		5♣, Q♦	2	His Heels, since the starter is
		5♣, J♣	2	a Jack.
	runs	4♥, 5♣, 6♥	3	
		Total	9 points	
Finally:		6♦, 6♥, 6♣, J♠.		Starter is 3♠.
	15s	6♦, 6♥, 3♠	2	Do you see how each 6 takes
		6♥, 6♦, 3♠	2	it's turn to score with the
		6♦, 6♣, 3♠	2	other cards?
	pairs	6♥, 6♦	2	The starter makes the J♠
		6♥, 6♣	2	worth an extra point here.
		6♦, 6♣	2	
	1 for His Nob J♠		1	
		Total	13 points	

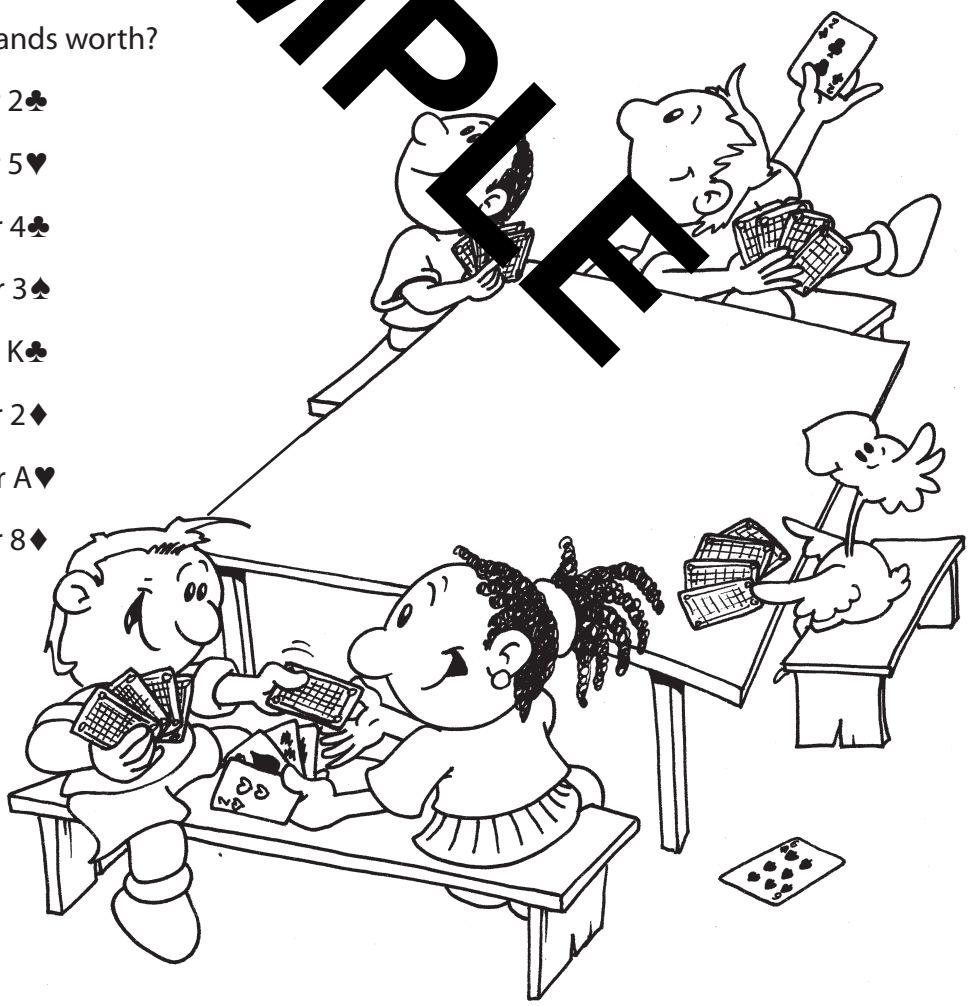
SAMPLE

** Using Your Skills

Your turn!

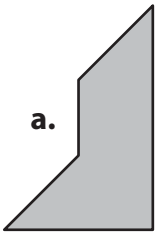
What are these cribbage hands worth?

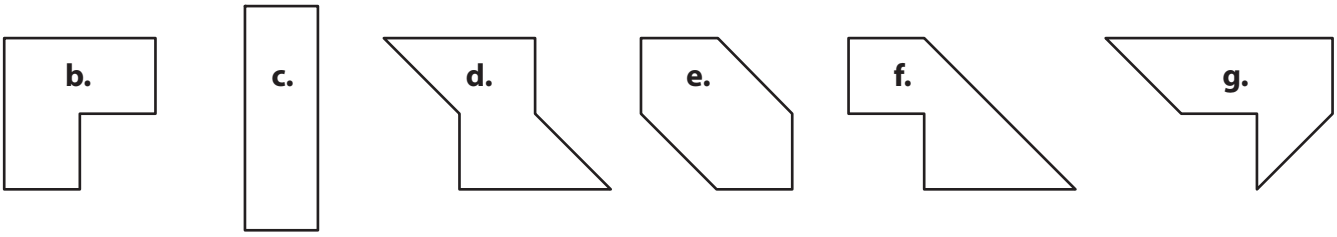
1. 3♣, 4♦, K♥, 8♣, starter 2♣
2. Q♥, 8♦, J♦, 5♠, starter 5♥
3. A♠, 3♠, 5♥, 9♣, starter 4♣
4. K♦, Q♦, J♦, A♥, starter 3♠
5. 2♦, 4♦, 6♥, 8♠, starter K♣
6. 6♠, 7♣, 7♦, A♥, starter 2♦
7. A♣, A♠, 6♦, 5♥, starter A♥
8. 6♦, 8♣, A♥, 9♠, starter 8♦



*** Two Easy Pieces

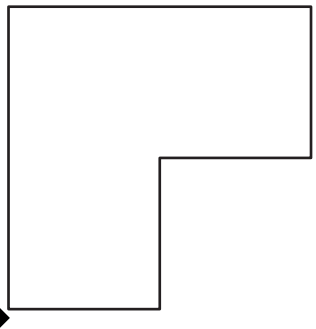
Cut out figure A. Then cut it into two pieces which can be used to make up each of the other figures below.

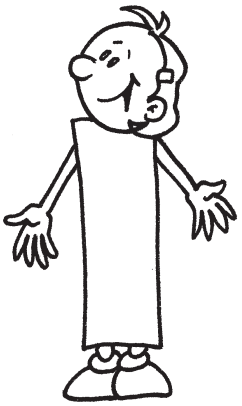




*** Four Easy Pieces

Show how this figure can be cut into four identical smaller pieces.





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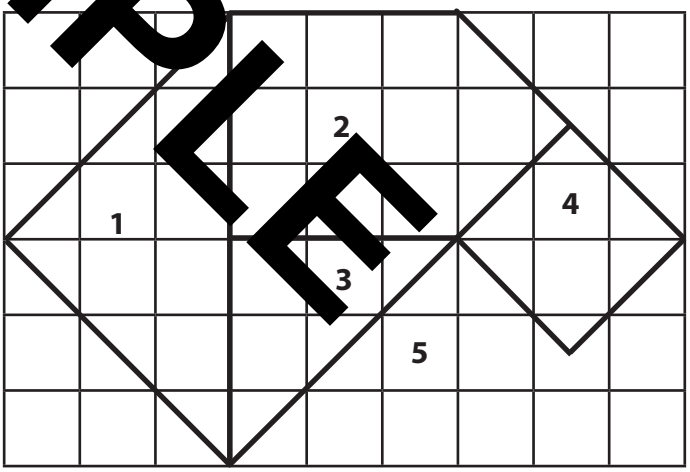
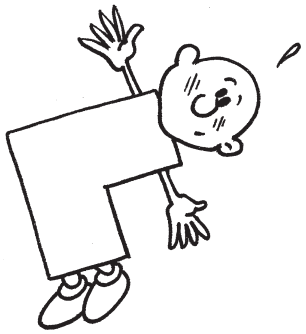
*** Five Easy Pieces

Carefully cut out the five numbered pieces of this figure. Then show how they can all be fitted together to form a square. It is tricky, but it can be done!





*** Five More Easy Pieces

Show how this square can be cut into five pentominoes, so that each one contains each of the numbers 1 to 5 exactly once each.

Can you design a different one for yourself? Try it.

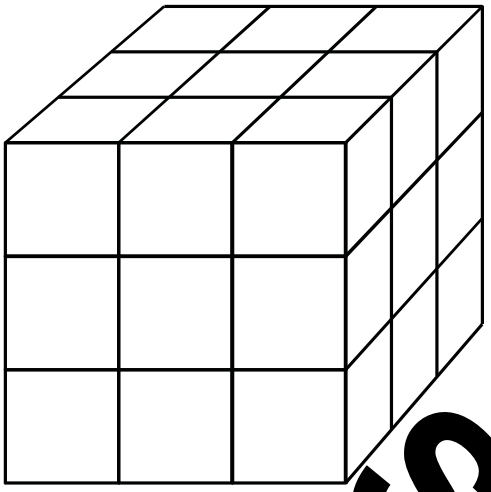
2	1	3	4	3
5	2	5	2	4
4	3	1	4	1
3	5	2	1	3
1	4	5	2	5

Face Painting

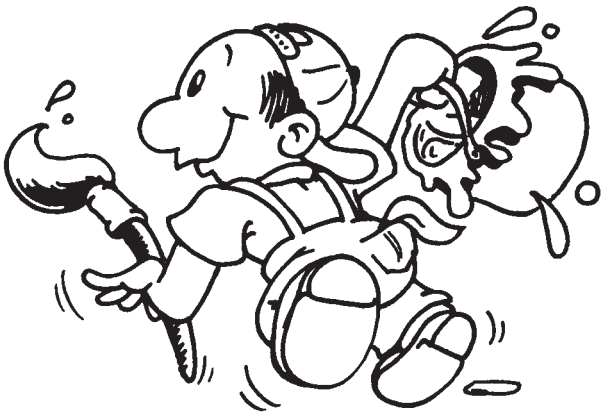
Investigation 8

* Finding Out

Fred built a cube containing 3 × 3 × 3 cubes each measuring 1 cm. There are 27 cubes altogether.



Then Fred painted the 6 outside faces of the large cube.



If Fred then separated the 27 cubes and sorted them, how many have …

0 faces painted _____

1 face painted _____

2 faces painted _____

3 faces painted _____

SAMPLE

** Using Your Skills

Suppose Fred now built a 4 × 4 × 4 cube and painted the outside faces of it. Draw it here. When he sorted the small cubes, he would find:

_____ cubes with 0 faces painted.

_____ cubes with 1 face painted.

_____ cubes with 2 faces painted.

_____ cubes with 3 faces painted.

[Check: how many cubes altogether?]

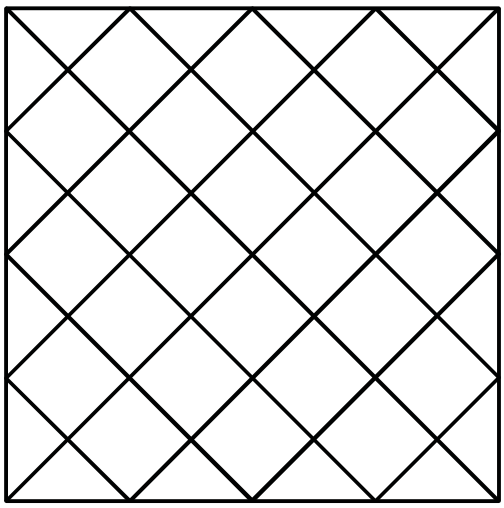
*** Taking It Further

EXTRA: See if you can work out the numbers for a 5 × 5 × 5 cube!

You may like to draw it on another sheet of paper.

*** Taking It Further

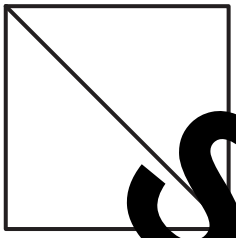
Sometimes tiles are laid diagonally to give an attractive diamond effect, like this.



The edge tiles are all cut on the diagonal to fit the wall.

If each tile measures 30 × 30 cm, how long will the cut edge of one tile be, to the nearest half centimetre?

To help you work this out measure the diagonal of this 3 cm square, to 2 decimal places if possible.



It is _____ cm. Therefore the cut edge of a 30 × 30 cm tile would be _____

The grid below measures 10 cm × 10 cm. It represents an area of floor 1 m × 1 m in size.

Carefully measure where to place the 30 cm × 30 cm tiles on this grid in a diagonal pattern, starting at one corner. Begin by drawing one diagonal.

SAMPLE